The physics of electrokinetic devices applying and adapting the Child-Langmuir Law derivation for vacuum diodes

Part 2: electrokinetic devices in air

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Recommended Reading

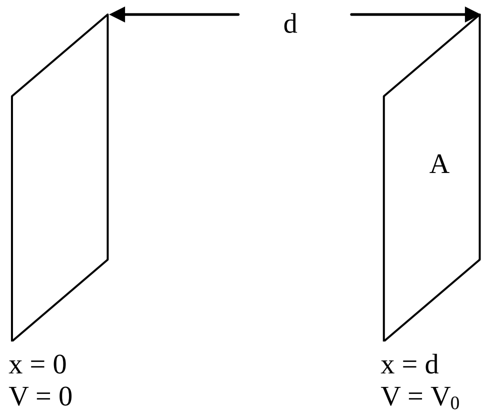
'Introduction to Electrodynamics' by David J. Griffiths. This and other related physics textbooks can be purchased here: Amazon Electrokinetic Physics Books Links

Introduction

The Child-Langmuir Law describes the characteristics of a parallel plate vacuum diode. By using this approach, we can derive a one dimensional expression for the characteristic properties of Lifters and related electrokinetic devices and by drawing an analogy to the vacuum diode, gain insights into the Lifter's subtle properties. A possible explanation for the Biefeld-Brown effect is given for both the air and vacuum.

Derivation of the characteristic equations for a electrokinetic device in air

Let us approximate our electrokinetic device as 2 plates, one plate grounded and the other at $V = V_0$. Let them be a distance 'd' apart and let us assume they are very large so that we can neglect end effects (Area = A >> d^2). Therefore, all properties will only depend on 'x' and we can make a 1-D approximation.



We know:

$$\frac{d^2V}{dx^2} = -\frac{\rho(x)}{\varepsilon_0}$$ (Poisson's equation) (1)

$E = -\frac{dV}{dx}$ (Definition of potential) (2)

$v(x) = \kappa E(x)$ (Blanc's Law for the mobility of ions in a medium) (3)

NB: We have now departed from our vacuum derivation in that the motion of each electron is defined by a new equation due to the influence of air on the motion of the ions formed.

Also
$I = -jA = -\rho v A$(Definition of current) (4)

Combining (1) and (4) we get

$$\frac{d^2V}{dx^2} = \frac{I}{v(x)A\varepsilon_0} \quad (5)$$

Combining this with (3) yields

$$\frac{d^2V}{dx^2} = \frac{I}{\kappa E(x)A\varepsilon_0} \quad (6)$$

Combining this with (2) yields

$$\frac{d^2V}{dx^2} = -\frac{I}{\kappa \frac{dV}{dx} A\varepsilon_0} \quad (7)$$

Rearranging yields

$$\frac{dV}{dx}\frac{d^2V}{dx^2} = \frac{1}{2}\frac{d}{dx}\left(\frac{dV}{dx}\right)^2 = -\frac{I}{\kappa A\varepsilon_0} \quad (8)$$

Rearranging and integrating yields

$$\left(\frac{dV}{dx}\right)^2 = -\frac{2I}{\kappa A\varepsilon_0}x + C \quad (9)$$

Assuming at x = 0, E = 0, we get

$$\frac{dV}{dx} = \left(-\frac{2I}{\kappa A\varepsilon_0}x\right)^{1/2} = \left(-\frac{2I}{\kappa A\varepsilon_0}\right)^{1/2} x^{1/2} \quad (10)$$

Integrating by x yields

$$V(x) = \left(-\frac{2I}{\kappa A\varepsilon_0}\right)^{1/2}\frac{2}{3}x^{3/2} + C \quad (11)$$

At x = 0, V = 0 so

$$V(x) = \left(-\frac{2I}{\kappa A\varepsilon_0}\right)^{1/2}\frac{2}{3}x^{3/2} = \left(-\frac{8}{9\kappa A\varepsilon_0}\right)^{1/2} I^{1/2}x^{3/2} \quad (12)$$

At x = d, $V = V_0$, therefore taking (12) and rearranging for I, we get

$$I = V_0^{\,2} d^{-3}\left(-\frac{8}{9\kappa A\varepsilon_0}\right)^{-1} = \frac{V_0^{\,2}}{d^3}\left(-\frac{9\kappa A\varepsilon_0}{8}\right) (13)$$

This is the air equivalent of the Child-Langmuir law. The current goes with the square of the potential and inversely with the cube of the gap length.

We also obtain

$$V(x) = \left(-\frac{2}{\kappa A\varepsilon_0}\frac{V_0^{\,2}}{d^3}\left(-\frac{9\kappa A\varepsilon_0}{8}\right)\right)^{1/2}\frac{2}{3}x^{3/2} = \left(\frac{9}{4d^3}\right)^{1/2} V_0\frac{2}{3}x^{3/2} = V_0 d^{-3/2}x^{3/2} (14)$$

Combining (2) and (14) yields

$$E(x) = -\frac{3}{2}V_0 d^{-3/2}x^{1/2} (15)$$

Combining (3) and (15) yields

$$v(x) = -\frac{3}{2}V_0 \kappa d^{-3/2}x^{1/2} (16)$$

Combining (1), (2) and (15) yields

$$\rho(x) = \varepsilon_0\frac{dE}{dx} = -\varepsilon_0\frac{3}{2}V_0 d^{-3/2}\frac{1}{2}x^{-1/2} = -\varepsilon_0\frac{3}{4}V_0 d^{-3/2}x^{-1/2} (17)$$

The force on the electrons in the gap is defined by the Lorentz force law for electrostatic charges

$$F(x) = A\rho(x)E(x) (18)$$

Combining this with (15) and (17) yields

$$F(x) = A\rho(x)E(x) = A\varepsilon_0\frac{3}{4}V_0 d^{-3/2}x^{-1/2}\frac{3}{2}V_0 d^{-3/2}x^{1/2} = A\varepsilon_0\frac{9}{8}V_0^{\,2}d^{-3} (18)$$

What is interesting about this result is that it is independent of x. Integrating over the gap we get

$$F_T = \int_0^d F(x)dx = AV_0^{\,2}\varepsilon_0\frac{9}{8}d^{-2} (19)$$

Combining (13) and (19) we get

$$F_T = A\varepsilon_0\frac{9}{8}d^{-2}Id^3\left(-\frac{9\kappa A\varepsilon_0}{8}\right)^{-1} = -\frac{Id}{\kappa} (20)$$

Given the force on the device will be equal and opposite to the force on the ions in the gap, we obtain our familiar form of

$$F_{EK} = \frac{Id}{\kappa} (21)$$

For completeness, we can also calculate the opposing force due to the change in momentum.

$$F(d)=\frac{d(mv)}{dt}=mv\frac{dv}{dx}=m\frac{3}{2}V_0\kappa d^{-3/2}x^{1/2}\frac{3}{2}V_0\kappa d^{-3/2}\frac{1}{2}x^{-1/2}=m\frac{9}{8}V_0{}^2\kappa^2 d^{-3}\quad(22)$$

Therefore our total force on the device is really the difference of (21) and (22), that is

$$F_{Final}=m\frac{9}{8}V_0{}^2\kappa^2 d^{-2}-AV_0{}^2\varepsilon_0\frac{9}{8}d^{-2}=\frac{9}{8}\left(\frac{V_0}{d}\right)^2\left(\frac{m\kappa^2}{d}-A\varepsilon_0\right)\quad(23)$$

Consequences for electrokinetic devices

While ions are formed in the medium surrounding the electrokinetic device, the Biefeld-Brown effect can be explained in terms of a loss of momentum to the medium through collisions between the air and the ions. Simply put, while in a vacuum the pull on the device from the charge-driven force exactly cancels the mass-driven movement when the electron is collected, for air, some of the ion's momentum is already diminished through collisions with the air so its impact with the collector is correspondingly reduced. The net effect will be an observed force towards the emitting electrode with no directly observable mechanism, what is often referred to erroneously as the 'unbalanced force'.

Comparison of experimental observation to the model

While a literature search of the major peer-reviewed journals resulted in little evidence of experimentation with electrokinetic devices, a number of sites exist on the internet where experimental results have been published. While not peer-reviewed, it is still instructive to determine where reality and the model coincide and where they do not. Any reported experiment which is inconsistent with this model can be replicated and gives direction for refining the model.

Experimental Observation	**Source**	**Consistent with proposed model?**	**Details**
Electrokinetic devices show movement in air	Multiple. http://www.blazelabs.com/e-exp04.asp http://jlnlabs.imars.com/lifters/logbook/index.htm	Yes	The model predicts a force in air
Increasing the temperature of the emitter, increases emitter current and thrust	http://www.blazelabs.com/e-exp07.asp	Yes	For a fixed gap, the force is proportional to the current. Therefore an increase in current will lead to an increase in observed force
Making the electrodes out of different materials affects the observed force	http://www.blazelabs.com/e-exp08.asp	No	The model makes no comment on the materials used for the electrodes and therefore they should not affect the observed force
Measured force is dependant on the medium the	http://www.blazelabs.com/e-exp12.asp	Yes	The ion mobility constant, κ, is dependant on the

device is operating in			medium the ion is travelling through so this is consistent
Electrokinetic devices stop working when put in a vacuum	http://www.blazelabs.com/l-vacuum.asp http://www.blazelabs.com/nasatest.pdf	Yes	In Part one, we predict that if no current is lost to the vacuum container, there will be no net force observed
Wire polarity affects the force	http://www.blazelabs.com/l-doe.asp	Yes	The ion mobility constant is dependant on the type of ions formed which depends on the polarity
There is an asymmetric force. The force on the emitter is different to the collector	http://jlnlabs.imars.com/lifters/asymmetric/index.htm	Yes	As the field at the emitter is zero, the induced charge on the emitter is also zero. However, as there is a field on the collector, there is also an induced charge on the collector. As F=qE, this means the collector experiences a force but the emitter does not. This logic also applies to the vacuum i.e. if there is current loss in the vacuum, the force will act on the collector, not the emitter

Areas for further research

For a given working device, it should be relatively simple to adjust the key parameters and measure the effect on the other measurable parameters i.e. plot the IV curve, determine the relationship between gap, voltage, current and observed force. This can then be compared to the relationships predicted by this model and the model's effectiveness can be assessed.

While vacuum tests appear to confirm an electrokinetic device will not work in a vacuum, quantitative tests of electrokinetic device performance at different air pressures will yield information on how performance is affected by, for instance, atmospheric and weather changes.

In one experiment it was suggested that the electrode material can affect performance. This is a remarkable result and if the specific quality of the material which affects electrokinetic performance can be identified, this could be exploited for better performance.

Finally, while experiments have been performed in different gases, the model also allows for a measurable force in dielectric liquids. Experiments could be performed on different liquids, including water to see how the force is affected. While the force in the air is quite weak, it may be discovered that the electrokinetic force could be employed for other purposes such as liquid transport or underwater propulsion.

Acknowledgements

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